

R22

Code No: 181AN

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year I Semester Examinations, September - 2023

MATRICES AND CALCULUS

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, AE, MIE, CSIT, CE(SE), CSE(CS), CSE(AI&ML), CSE(DS), CSE(IOT), AI&DS, AI&ML,CSD)

Time: 3 Hours

Max. Marks: 60

Note: This question paper contains two parts A and B.

i) Part- A for 10 marks, ii) Part - B for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of ten questions (numbered from 2 to 11) carrying 10 marks each. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART - A

(10 Marks)

- 1.a) Define rank of a matrix. [1]
- b) What do you mean by indirect methods? [1]
- c) What is characteristic equation of a matrix? [1]
- d) Write the symmetric matrix in the matrix notation of the quadratic form:  
 $2xy + 2yz + 2zx$ . [1]
- e) State Lagrange's mean value theorem. [1]
- f) Define improper integral. [1]
- g) State Euler's theorem. [1]
- h) What are stationary points of  $z = f(x, y)$ ? [1]
- i) Define volume integral of a vector point function  $\vec{F}$  over a region V. [1]
- j) For finding the area of a plane region R, which double integral is to be evaluated. [1]

PART - B

(50 Marks)

2.a) Determine the rank of the matrix by reducing it into normal form

$$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

b) Apply Gauss elimination method to solve the equations  
 $x + 4y - z = -5$ ;  $x + y - 6z = -12$ ;  $3x - y - z = 4$ . [5+5]

OR

- 3.a) Apply Gauss -Jordan method to solve the equations:  
 $2x + 3y - z = -1$ ,  $x + 4y + 5z = 25$ ,  $3x - 4y + z = 2$ .
- b) Apply Gauss -Siedel method to solve the equations:  
 $2x + y + 6z = 9$ ,  $8x + 3y + 2z = 13$ ,  $x + 5y + z = 7$ . [5+5]

- 4.a) Find the Eigen values and Eigen vectors of the following matrix  $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 1 & -4 & 3 \end{bmatrix}$ .

- b) Using Cayley-Hamilton theorem the matrix  $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$  compute  $A^{-1}$ . Also find for  $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ . [5+5]

OR

- 5.a) Using Cayley Hamilton theorem for the matrix  $\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$  and find its inverse.

- b) Reduce the quadratic form  $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4zx$  into a sum of squares by an orthogonal transformation and also discuss its nature. [5+5]

- 6.a) Verify Rolle's theorem for the function  $f(x) = \log_e x$  in  $[1, e]$ .

- b) Expand  $f(x, y) = \sin xy$  in powers of  $(x-1)$  and  $(y-\pi/2)$  upto second degree terms.

[5+5]

OR

- 7.a) Find the volume of the revolution of a loop of the curve  $y^2(x+b) - x^2(3b-x) = 0$  about  $x$ -axis.

- b) Prove that  $\Gamma(m)\Gamma(m + 1/2) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m)$ . [5+5]

- 8.a) If  $u = \operatorname{cosec}^{-1} \left( \frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}} \right)^{1/2}$  then prove that  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12} \left[ \frac{13}{12} + \frac{\tan^2 u}{12} \right]$ .

- b) If  $u = \frac{x+y}{1-xy}$  and  $v = \tan^{-1} x + \tan^{-1} y$ , find  $\frac{\partial(u,v)}{\partial(x,y)}$ . Are  $u$  and  $v$  functionally related? If so find its relation. [5+5]

OR

- 9.a) Find the maximum and minimum values of  $x^3 y^2 (1-x-y)$ .

- b) A rectangular box open at top is to have a volume of 32 cubic feet. Find the dimensions of the box requiring least material for its construction. [5+5]

- 10.a) Evaluate  $\iint xy dx dy$  over the positive quadrant of the circle  $x^2 + y^2 = a^2$ .

- b) Evaluate the integral  $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$  by changing the order of integration. [5+5]

OR

- 11.a) Find by double integration the area of the lemniscate  $r^2 = a^2 \cos 2\theta$ .

- b) Find the volume bounded by the cylinder  $x^2 + y^2 = 4$  and the planes  $y+z=4$  and  $z=0$ . [5+5]